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# STUDY PACKAGE

## Subject : Mathematics

## Topic : LIMITS

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# Limit

## 1. Limit of a function f(x) is said to exist as, $x \to a$ when,

$$\underset{h \to 0^+}{\text{Limit}} f(a - h) = \underset{h \to 0^+}{\text{Limit}} f(a + h) = \text{some finite value M.}$$

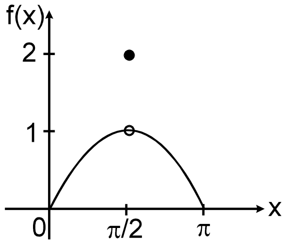
(Left hand limit) (Right hand limit)

Note that we are not interested in knowing about what happens at x = a. Also note that if L.H.L. & R.H.L. are both tending towards '$\infty$' or '$-\infty$' then it is said to be infinite limit.

Remember, $\underset{x \to a}{\text{Limit}} \Rightarrow x \neq a$

**Solved Example # 1**

Find $\underset{x \to \pi/2}{\text{Limit}} f(x)$

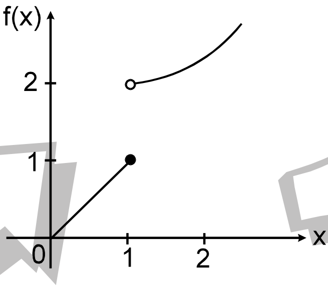



**Solution.**

Here $\underset{x \to \pi/2}{\text{Limit}} f(x) = 1$

**Solved Example # 2**

Find $\underset{x \to 1}{\text{Limit}} f(x)$



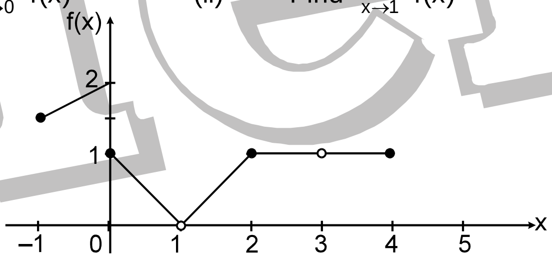

**Solution.**

Left handed limit = 1 Right handed limit = 2

Hence $\underset{x \to 1}{\text{Limit}} f(x)$ = does not exist.

**Solved Example # 3**

(i) Find $\underset{x \to 0}{\text{limit}} f(x)$ (ii) Find $\underset{x \to 1}{\text{limit}} f(x)$ (iii) Find $\underset{x \to 3}{\text{limit}} f(x)$




**Solution.**

(i) $\underset{x \to 0}{\text{limit}} f(x)$ = does not exists
because left handed limit $\neq$ right handed limit

(ii) $\underset{x \to 1}{\text{limit}} f(x) = 0$ (iii) $\underset{x \to 3}{\text{limit}} f(x) = 1$

## 2. Indeterminant Forms:

$\frac{0}{0}, \frac{\infty}{\infty}, 0 \times \infty, \infty - \infty, \infty^0, 0^0$, and $1^\infty$.

**Solved Example # 4**

Which of the following limits are forming indeterminant from also indicate the form

(i) $\lim\limits_{x \to 0} \frac{1}{x}$ (ii) $\lim\limits_{x \to 0} \frac{1 - x}{1 - x^2}$

(iii) $\lim\limits_{x \to 0} x \, \ell n \, x$ (iv) $\lim\limits_{x \to 0} \left(\frac{1}{x} - \frac{1}{x^2}\right)$

(v) $\lim_{x\to 0} (\sin x)^x$ (vi) $\lim_{x\to 0} (\ell n\, x)^x$

(vii) $\lim_{x\to 0} (1+\sin x)^{\frac{1}{x}}$ (viii) $\lim_{x\to 0} (1)^{1/x}$

**Solution**

(i) No (ii) Yes $\frac{0}{0}$ from

(iii) Yes $0 \times \infty$ form (iv) Yes $(\infty - \infty)$ form

(v) Yes, $(0)^o$ form (vi) Yes $(\infty)^o$ form

(vii) Yes $(1)^\infty$ form (viii)

**NOTE :**

(i) ' 0 ' doesn't means exact zero but represent a value approaching towards zero similary to ' 1 ' and infinity.

(ii) $\infty + \infty = \infty$

(iii) $\infty \times \infty = \infty$

(iv) $(a/\infty) = 0$ if a is finite

(v) $\frac{a}{0}$ is not defined for any $a \in R$.

(vi) a b = 0, if & only if a = 0 or b = 0 and a & b are finite.

## 3. Method of Removing Indeterminancy

To evaluate a limit, we must always put the value where ' x ' is approaching to in the function. If we get a determinate form, then that value becomes the limit otherwise if an indeterminant form comes. Then apply one of the following methods:

(i) Factorisation (ii) Rationalisation or double rationalisation

(iii) Substitution (iv) Using standard limits

(v) Expansions of functions.

**1. Factorization method :-**

We can cancel out the factors which are leading to indeterminancy and find the limit of the remaing expression.

**Solved Example # 5**

$$\underset{x\to 2}{\text{limit}} \frac{x^6 - 24x - 16}{x^3 + 2x - 12}$$

**Solution.**

$$\underset{x\to 2}{\text{limit}} \frac{x^6 - 24x - 16}{x^3 + 2x - 12} = \underset{x\to 2}{\text{limit}} \frac{(x-2)(x^5 + 2x^4 + 4x^3 + 8x^2 + 16x + 8)}{(x^2 + 2x + 6)(x-2)}$$

$$= \frac{168}{14} = \mathbf{12}$$

**2. Rationalization /Double Rationalization.**

We can rationalize the irrational expression by multiplying with their conjugates to remove the indeterminancy.

**Solved Example # 6** $\underset{x\to 1}{\text{limit}} \frac{4-\sqrt{5x+1}}{2-\sqrt{3x+1}}$.

**Solution.** $\underset{x\to 1}{\text{limit}} \frac{4-\sqrt{5x+1}}{2-\sqrt{3x+1}}$

$$= \underset{x\to 1}{\text{limit}} \frac{(4-\sqrt{5x+1})(2+\sqrt{3x+1})(4+\sqrt{5x+1})}{(2-\sqrt{3x+1})(4+\sqrt{5x+1})(2+\sqrt{3x+1})}$$

$$= \underset{x\to 1}{\text{limit}} \frac{(15-5x)}{(3-3x)} \times \frac{2+\sqrt{3x+1}}{4+\sqrt{5x+1}} = \frac{5}{6}$$

**Solved Example # 7**

Evaluate : (i) $\lim_{x\to 2} \left[\frac{1}{x-2} - \frac{2\,(2x-3)}{x^3 - 3x^2 + 2x}\right]$ (ii) $\lim_{x\to 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x}$

(iii) $\lim_{x\to 1} \frac{(2x-3)\left(\sqrt{x}-1\right)}{2x^2 + x - 3}$

**Solution** (i) We have

$$\lim_{x\to 2} \left[\frac{1}{x-2} - \frac{2\,(2x-3)}{x^3 - 3x^2 + 2x}\right] = \lim_{x\to 2} \left[\frac{1}{x-2} - \frac{2\,(2x-3)}{x(x-1)(x-2)}\right]$$

$$= \lim_{x\to 2} \left[\frac{x(x-1) - 2(2x-3)}{x(x-1)(x-2)}\right]$$



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$$= \lim_{x\to2}\left[\frac{x^2-5x+6}{x(x-1)(x-2)}\right]$$

$$= \lim_{x\to2}\left[\frac{(x-2)(x-3)}{x(x-1)(x-2)}\right] \qquad = \lim_{x\to2}\left[\frac{x-3}{x(x-1)}\right] = -\frac{1}{2}$$

(ii) The given limit taken the form $\frac{0}{0}$ when $x \to 0$. Rationalising the numerator, we get

$$\lim_{x\to0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x} = \lim_{x\to0}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{x}\times\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]$$

$$= \lim_{x\to0}\left[\frac{(1+x)-(1-x)}{x\left(\sqrt{1+x}+\sqrt{1-x}\right)}\right]$$

$$= \lim_{x\to0}\left[\frac{2x}{x\left(\sqrt{1+x}+\sqrt{1-x}\right)}\right] \qquad = \lim_{x\to0}\left[\frac{2}{\sqrt{1+x}+\sqrt{1-x}}\right] = \frac{2}{2} = 1$$

(iii) We have

$$\lim_{x\to1}\left[\frac{(2x-3)\left(\sqrt{x}-1\right)}{2x^2+x-3}\right] = \lim_{x\to1}\left[\frac{(2x-3)\left(\sqrt{x}-1\right)}{(2x+3)(x-1)}\right]$$

$$= \lim_{x\to1}\left[\frac{(2x-3)\left(\sqrt{x}-1\right)}{(2x+3)\left(\sqrt{x}-1\right)\left(\sqrt{x}+1\right)}\right]$$

$$= \lim_{x\to1}\left[\frac{2x-3}{(2x+3)\left(\sqrt{x}+1\right)}\right] \qquad = \frac{-1}{(5)(2)} = \frac{-1}{10}$$

## 4. Fundamental Theorems on Limits:

Let $\underset{x\to a}{\text{Limit}}\ f(x) = \ell$ & $\underset{x\to a}{\text{Limit}}\ g(x) = m$. If $\ell$ & m exists then:

(i) $\underset{x\to a}{\text{Limit}}\ \{f(x) \pm g(x)\} = \ell \pm m$ (ii) $\underset{x\to a}{\text{Limit}}\ \{f(x).\ g(x)\} = \ell.m$

(iii) $\underset{x\to a}{\text{Limit}}\ \frac{f(x)}{g(x)} = \frac{\ell}{m}$, provided $m \neq 0$

(iv) $\underset{x\to a}{\text{Limit}}\ k f(x) = k\ \underset{x\to a}{\text{Limit}}\ f(x)$ ; where k is a constant.

(v) $\underset{x\to a}{\text{Limit}}\ f[g(x)] = f\left(\underset{x\to a}{\text{Limit}}\ g(x)\right) = f(m)$; provided f is continuous at g (x) = m.

**Solved Example # 8** Evaluate

(i) $\lim_{x\to2}(x+2)$ (ii) $\lim_{x\to2}x(x-1)$ (iii) $\lim_{x\to2}\frac{x^2+4}{x+2}$ (iv) $\lim_{x\to0}\cos(\sin x)$

(v) $\lim_{x\to1}\frac{x^2-3x+2}{x^2-1}$ (vi) $\lim_{x\to1}\frac{x^2+3x+2}{x^2-1}$

**Solution** (i) x + 2 being a polynomial in x, its limit as $x \to 2$ is given by $\lim_{x\to2}(x+2) = 2 + 2 = 4$

(ii) Again x(x – 1) being a polynomial in x, its limit as $x \to 2$ is given by

$$\lim_{x\to2}x(x-1) = 2(2-1) = 2$$

(iii) By (II) above, we have $\lim_{x\to2}\frac{x^2+4}{x+2} = \frac{(2)^2+4}{2+2} = 2$

(iv) $\lim_{x\to0}\cos(\sin x) = \cos\left(\lim_{x\to0}\sin x\right) = \cos 0 = 1$

(v) Note that for x = 1 both the numerator and the denominator of the given fraction vanish. Therefore by (III) above, we have $\lim_{x\to1}\frac{x^2-3x+2}{x^2-1} = \lim_{x\to1}\frac{(x-1)(x-2)}{(x-1)(x+1)} = \lim_{x\to1}\frac{x-2}{x+1} = -\frac{1}{2}$

(vi) Note that for x = 1, the numerator of the given expression is a non-zero constant 6 and the denominator is zero. Therefore, the given limit is of the form $\frac{6}{0}$. Hence, by (IV) above, we conclude that $\lim_{x\to1}\frac{x^2+3x+2}{x^2-1}$ does not exist

## 5. Standard Limits:

(a) $\underset{x\to0}{\text{Limit}}\ \frac{\sin x}{x} = 1 = \underset{x\to0}{\text{Limit}}\ \frac{\tan x}{x} = \underset{x\to0}{\text{Limit}}\ \frac{\tan^{-1}x}{x} = \underset{x\to0}{\text{Limit}}\ \frac{\sin^{-1}x}{x}$

[ Where x is measured in radians ]

(b) $\underset{x\to0}{\text{Limit}}\ (1+x)^{1/x} = e$ ; $\underset{x\to\infty}{\text{Limit}}\ \left(1+\frac{1}{x}\right)^x = e$

(c) $\underset{x\to 0}{\text{Limit}}\ \frac{e^x - 1}{x} = 1;$ $\underset{x\to 0}{\text{Limit}}\ \frac{a^x - 1}{x} = \log_e a,\ a > 0$

(d) $\underset{x\to 0}{\text{Limit}}\ \frac{\ell n(1+x)}{x} = 1$ (e) $\underset{x\to a}{\text{Limit}}\ \frac{x^n - a^n}{x - a} = na^{n-1}$.

**Solved Example # 9:** Find $\underset{x\to 0}{\text{Limit}}\ \frac{\sin 2x}{x}$

**Solution.** $\underset{x\to 0}{\text{Limit}}\ \frac{\sin 2x}{x} \Rightarrow \underset{x\to 0}{\text{Limit}}\ \frac{\sin 2x}{2x} . 2 = 2$

**Solved Example # 10:** $\underset{x\to 0}{\text{Limit}}\ \frac{e^{3x} - 1}{x/2}$

**Solution.** $\underset{x\to 0}{\text{Limit}}\ \frac{e^{3x} - 1}{x/2}$ $\underset{x\to 0}{\text{Limit}}\ 2 \times 3\ \frac{e^{3x} - 1}{3x} = -6.$

**Solved Example # 11** $\underset{x\to 0}{\text{Limit}}\ \frac{\tan x - \sin x}{x^3}$

**Solution.** $\underset{x\to 0}{\text{Limit}}\ \frac{\tan x - \sin x}{x^3}$

$= \underset{x\to 0}{\text{Limit}}\ \frac{\tan x(1 - \cos x)}{x^3}$

$= \underset{x\to 0}{\text{Limit}}\ \frac{\tan x\ .\ 2\sin^2 \frac{x}{2}}{x^3} = \underset{x\to 0}{\text{Limit}}\ \frac{\tan x}{x}\ .\ \left(\frac{\sin \frac{x}{2}}{\frac{x}{2}}\right)^2 = 1.$

**Solved Example # 12** Compute $\lim_{x\to 0} \frac{\sin 2x}{\sin 3x}$

**Solution** We have $\lim_{x\to 0} \frac{\sin 2x}{\sin 3x} = \lim_{x\to 0} \left[\frac{\sin 2x}{2x} . \frac{2x}{3x} . \frac{3x}{\sin 3x}\right]$

$= \left[\lim_{2x\to 0} \frac{\sin 2x}{2x}\right] . \frac{2}{3} . \left[\lim_{3x\to 0} \frac{3x}{\sin 2x}\right],\ x \neq 0$

$= 1 . \frac{2}{3} + \left[\lim_{3x\to 0} \frac{\sin 3x}{3x}\right] = \frac{2}{3} \times 1 = \frac{2}{3}$

**Solved Example # 13** Evaluate $\lim_{x\to\infty} \left(1 + \frac{2}{x}\right)^x$

**Solution** $\lim_{x\to\infty} \left(1 + \frac{2}{x}\right)^x = e^{\lim_{x\to\infty} \frac{2}{x}.x} = e^2.$

**Solved Example # 14** Compute (i) $\lim_{x\to 3} \frac{e^x - e^3}{x - 3}$ (ii) $\lim_{x\to 0} \frac{x(e^x - 1)}{1 - \cos x}$

**Solution** (i) Put y = x – 3. So, as $x \to 3$, $y \to 0$. Thus

$\lim_{x\to 3} \frac{e^x - e^3}{x - 3} = \lim_{y\to 0} \frac{e^{3+y} - e^3}{y}$

$= \lim_{y\to 0} \frac{e^3 . e^y - e^3}{y}$

$= e^3 \lim_{y\to 0} \frac{e^y - 1}{y} = e^3 . 1 = e^3$

(ii) We have

(ii) $\lim_{x\to 0} \frac{x(e^x - 1)}{1 - \cos x} = \lim_{x\to 0} \frac{x(e^x - 1)}{2\sin^2 \frac{x}{2}}$

$= \frac{1}{2} . \lim_{x\to 0} \left[\frac{e^x - 1}{x} . \frac{x^2}{\sin^2 \frac{x}{2}}\right] = 2.$

**Solved Example # 15** Evaluate $\lim_{x\to 2} \frac{x^3 - 8}{x^2 - 4}$

**Solution** **(First Method)**

The given expression is of the form

$\frac{x^3 - (2)^3}{x^2 - (2)^2} = \frac{x^3 - (2)^3}{x - 2} \div \frac{x^2 - (2)^2}{x - 2}$



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$$\Rightarrow \quad \lim_{x\to 2}\frac{x^3-8}{x^2-4} = \lim_{x\to 2}\frac{x^3-(2)^3}{x-2} \div \lim_{x\to 2}\frac{x^2-(2)^2}{x-2}$$

$$= 3(2^2) \div 2(2^1) \qquad \left(\text{using } \lim_{x\to a}\frac{x^n-a^n}{x-a} = na^{n-1}\right)$$

$$= 12 \div 4 = 3$$

**(Second Method)**

The numerator and denominator have a common factor (x – 2). Cancelling this factor, we obtain

$$\frac{x^3-8}{x^2-4} = \frac{x^2+2x+4}{x+2} \quad \Rightarrow \quad \lim_{x\to 2}\frac{x^3-8}{x^2-4} = \lim_{x\to 2}\frac{x^2+2x+4}{x+2}$$

$$= \frac{(2)^2+2(2)+4}{2+2} = \frac{12}{4} = 3$$

Note : Since $x \to 2$, $x - 2$ is not zero, so the cancellation of the factor $x - 2$ in the above example is carried out.

## 6. Use of Subsitution in Solving Limit Problems

Sometimes in solving limit problem we convert $\lim_{x\to a} f(x)$ by subtituting $x = a + h$ or $x = a - h$ as $\lim_{h\to 0} f(a + h)$ or $\lim_{h\to 0} f(a - h)$ according as need of the problem.

**Solved Example # 16** $\quad \displaystyle \underset{x\to\pi/4}{\text{limit}} \frac{1-\tan x}{1-\sqrt{2}\sin x}$

**Solution.** Put $x = \frac{\pi}{4} + h \quad \because \quad x \to \frac{\pi}{4} \Rightarrow h \to 0$

$$\underset{h\to 0}{\text{limit}} \frac{1-\tan\left(\frac{\pi}{4}+h\right)}{1-\sqrt{2}\sin\left(\frac{\pi}{4}+h\right)}$$

$$= \underset{h\to 0}{\text{limit}} = \frac{1-\dfrac{1-\tan h}{1-\tan h}}{1-\sin h-\cos h}$$

$$= \underset{h\to 0}{\text{limit}} \frac{\dfrac{-2\tan h}{1-\tan h}}{2\sin^2\dfrac{h}{2}-2\sin\dfrac{h}{2}\cos\dfrac{h}{2}}$$

$$= \underset{h\to 0}{\text{limit}} \frac{-2\tan h}{2\sin^2\dfrac{h}{2}\left[2\sin\dfrac{h}{2}-\cos\dfrac{h}{2}\right]}\ \frac{1}{(1-\tan h)}$$

$$= \underset{h\to 0}{\text{limit}} \frac{-2\dfrac{\tan h}{h}}{\dfrac{\sin\frac{h}{2}}{\frac{h}{2}}\left[\sin\dfrac{h}{2}-\cos\dfrac{h}{2}\right]}\ \frac{1}{(1-\tan h)} = \frac{-2}{-1} = 2.$$

## 7. Limit When $x \to \infty$

Since $x \to \infty \Rightarrow \frac{1}{x} \to 0$ hence in this type of problem we express most of the part of expression in terms of $\frac{1}{x}$ and apply $\frac{1}{x} \to 0$. We can see this approch in the given solve examples.

**Solved Example # 17** $\quad \displaystyle \underset{x\to\infty}{\text{limit}}\ x \sin\frac{1}{x}$

**Solution.** $\displaystyle \underset{x\to\infty}{\text{limit}}\ x \sin\frac{1}{x}$

$$= \underset{x\to\infty}{\text{limit}} \frac{\sin 1/x}{1/x} = 1$$

**Solved Example # 18** $\quad \displaystyle \underset{x\to\infty}{\text{limit}} \frac{x-2}{2x-3}$

**Solution.** $\displaystyle \underset{x\to\infty}{\text{limit}} \frac{x-2}{2x-3}$

$$\underset{x\to\infty}{\text{limit}} \frac{1-2/x}{2-3/x} = \frac{1}{2}.$$

**Solved Example # 19** $\quad \displaystyle \underset{x\to\infty}{\text{limit}} \frac{x^2-4x+5}{3x^2-x^3+2}$





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**Solution.** $\lim\limits_{x\to\infty} \dfrac{x^2-4x+5}{3x^2-x^3+2}$

$$= \lim_{x\to\infty} \frac{\frac{1}{x}-\frac{4}{x^2}+\frac{5}{x^3}}{\frac{3}{x}-1+\frac{2}{x^3}} = 0$$

**Solved Example # 20** $\lim\limits_{x\to-\infty} \dfrac{\sqrt{3x^2+2}}{x-2}$

**Solution.** $\lim\limits_{x\to-\infty} \dfrac{\sqrt{3x^2+2}}{x-2}$

Put $x = \dfrac{-1}{t}$ $\quad x \to -\infty \quad t \to 0^+$

$$= \lim_{x\to 0^+} \frac{\sqrt{3+2t^2}\,.\,\frac{1}{\sqrt{t^2}}}{\frac{1-2t}{t}}$$

$$= \lim_{x\to 0^+} \frac{\sqrt{3+2t^2}}{-(1+2t)}\,\frac{t}{|t|} = \frac{\sqrt{3}}{-1} = -\sqrt{3}\,.$$

## 8. Limits Using Expansion

(i) $a^x = 1+\dfrac{x\ln a}{1!}+\dfrac{x^2\ln^2 a}{2!}+\dfrac{x^3\ln^3 a}{3!}+\dots\dots\dots a>0$ (ii) $e^x = 1+\dfrac{x}{1!}+\dfrac{x^2}{2!}+\dfrac{x^3}{3!}+\dots\dots$

(iii) $\ln(1+x) = x-\dfrac{x^2}{2}+\dfrac{x^3}{3}-\dfrac{x^4}{4}+\dots\dots\dots \text{for } -1<x\le 1$ (iv) $\sin x = x-\dfrac{x^3}{3!}+\dfrac{x^5}{5!}-\dfrac{x^7}{7!}+\dots\dots$

(v) $\cos x = 1-\dfrac{x^2}{2!}+\dfrac{x^4}{4!}-\dfrac{x^6}{6!}+\dots\dots$ (vi) $\tan x = x+\dfrac{x^3}{3}+\dfrac{2x^5}{15}+\dots\dots$

(vii) $\tan^{-1}x = x-\dfrac{x^3}{3}+\dfrac{x^5}{5}-\dfrac{x^7}{7}+\dots\dots$ (viii) $\sin^{-1}x = x+\dfrac{1^2}{3!}x^3+\dfrac{1^2.3^2}{5!}x^5+\dfrac{1^2.3^2.5^2}{7!}x^7+\dots\dots$

(ix) $\sec^{-1}x = 1+\dfrac{x^2}{2!}+\dfrac{5x^4}{4!}+\dfrac{61x^6}{6!}+\dots\dots$

(x) for $|x| < 1$, $n \in R$ $(1+x)^n = 1+nx+\dfrac{n(n-1)}{1.2}x^2+\dfrac{n(n-1)(n-2)}{1.2.3}x^3+\dots\dots\dots\dots\infty$

**Solved Example # 21** $\lim\limits_{x\to 0} \dfrac{e^x-1-x}{x^2}$

**Solution.** $\lim\limits_{x\to 0} \dfrac{e^x-1-x}{x^2}$

$$= \lim_{x\to 0} \frac{\left(1+x+\frac{x^2}{2!}\dots\dots\right)-1-x}{x^2} = \frac{1}{2}$$

**Solved Example # 22** $\lim\limits_{x\to 0} \dfrac{\tan x-\sin x}{x^3}$

**Solution.** $\lim\limits_{x\to 0} \dfrac{\tan x-\sin x}{x^3}$

$$= \lim_{x\to 0} \frac{\left(x+\frac{x^3}{3}\dots\dots\right)-\left(x-\frac{x^3}{3!}\dots\dots\right)}{x^3} = \frac{1}{3}+\frac{1}{6} = \frac{1}{2}.$$

**Solved Example # 23** $\lim\limits_{x\to 0} \dfrac{(7+x)^{1/3}-2}{x-1}$

**Solution.** Put $x \to 1+h$ $\lim\limits_{h\to 0} \dfrac{(8+h)^{1/3}-2}{h}$

$$\lim_{h\to 0} \frac{2.\left(1+\frac{h}{8}\right)^{1/3}-2}{h}$$

$$= \lim_{h\to 0} \frac{2\left\{1+\frac{1}{3}.\frac{h}{8}+\frac{\frac{1}{3}\left(\frac{1}{3}-1\right)\left(\frac{h}{8}\right)^2}{1.2}+\dots\dots-1\right\}}{h}$$



$$= \lim_{h \to 0} 2 \times \frac{1}{24} = \frac{1}{12}$$

**Solved Example # 24** $\lim_{x \to 0} \dfrac{\ell n(1+x) - \sin x + \frac{x^2}{2}}{x \tan x \sin x}$

$$\lim_{x \to 0} \frac{\ell n(1+x) - \sin x + \frac{x^2}{2}}{x \tan x \sin x}$$

$$= \lim_{x \to 0} \frac{\left(x - \frac{x^2}{2} + \frac{x^3}{3} .....\right) - \left(x + \frac{x^3}{3!} + \frac{x^5}{5!} + .....\right) + \frac{x^2}{2}}{x^3 . \frac{\tan x}{x} . \frac{\sin x}{x}} \qquad = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$$

## 9. Limits of form $1^\infty$, $0^0$, $\infty^0$

All these forms can be convered into $\frac{0}{0}$ form in the following ways

(i) If $x \to 1, y \to \infty$, then $z = (x)^y$

$\Rightarrow \quad \ell n\, z = y\, \ell n\, x \quad \Rightarrow \quad \ell n\, z = \dfrac{\ell n x}{(1/y)}$

Since $y \to \infty$ hence $\frac{1}{y} \to 0$ and $x \to 1$ hence $\ell nx \to 0$

(ii) If $x \to 0, y \to 0$, then $z = x^y \quad \Rightarrow \quad \ell n\, z = y\, \ell n\, x$

$\Rightarrow \quad \ell n\, z = \dfrac{y}{1/\ell ny} = \dfrac{0}{0}$ form

(iii) If $x \to \infty$, $y \to 0$, then $z = x^y \quad \Rightarrow \quad \ell n\, z = y\, \ell n\, x$

$\Rightarrow \quad \ell n\, z = \dfrac{y}{1/\ell nx} = \dfrac{0}{0}$ form

also for $(1)^\infty$ type of problems we can use following rules.

(i) $\lim_{x\to 0} (1 + x)^{1/x} = e$ (ii) $\lim_{x\to a} [f(x)]^{g(x)}$

where $f(x) \to 1$ ; $g(x) \to \infty$ as $x \to a$

$$= \lim_{x\to a} [1 + f(x) - 1]^{\frac{1}{f(x)-1}\{f(x)-1\}.g(x)} \qquad = e^{\lim_{x\to a}[f(x)-1]\, g(x)}$$

**Solved Example # 25** $\lim_{x\to\infty} \left(\dfrac{2x^2-1}{2x^2+3}\right)^{4x^2+2}$

**Solution.** Since it is in the form of $1^\infty$

$$\lim_{x\to\infty} \left(\frac{2x^2-1}{2x^2+3}\right)^{4x^2+2} = e^{\lim_{x\to\infty} \left(\frac{2x^2-1-2x^2-3}{2x^2+3}\right)(4x^2+2)} = e^{-8}$$

**Solved Example # 26**: $\lim_{x\to\frac{\pi}{4}} (\tan x)^{\tan 2x}$

**Solution** $= e^{\lim_{x\to\frac{\pi}{4}} (\tan x - 1)\tan 2x}$

$$= e^{\lim_{x\to\frac{\pi}{4}} (\tan x - 1)\frac{2\tan x}{1-\tan^2 x}}$$

$$= e^{2\times\frac{\tan \pi/4}{-1(1+\tan \pi/4)}} \qquad = e^{-1} = \frac{1}{e}$$

**Solved Example # 27** Evaluate $\lim_{x\to a}\left(2 - \dfrac{a}{x}\right)^{\tan\frac{\pi x}{2a}}$.

**Solution.** $\lim_{x\to a}\left(2 - \dfrac{a}{x}\right)^{\tan\frac{\pi x}{2a}}$

put $x = a + h \qquad \Rightarrow \qquad \lim_{h\to 0}\left(1 + \dfrac{h}{(a+h)}\right)^{\tan\left(\frac{\pi}{2}+\frac{\pi h}{2a}\right)}$

$\Rightarrow \quad \lim_{h\to 0}\left(1 + \dfrac{h}{a+h}\right)^{-\cot\left(\frac{\pi h}{2a}\right)} \qquad \Rightarrow \qquad e^{\lim_{h\to 0} -\cot\frac{\pi h}{2a}.\left(1+\frac{h}{a+h}-1\right)}$

$$\Rightarrow \quad e^{\lim_{h\to 0} -\left(\frac{\frac{\pi h}{2a}}{\tan\frac{\pi h}{2a}}\right).\frac{\frac{2a}{\pi}}{a+h}} \qquad = e^{-2/\pi}$$

**Solved Example # 28:** $\lim\limits_{x\to 0^+} x^x$

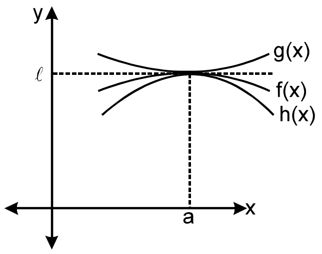
**Solution.** $y = \lim\limits_{x\to 0} x^x$

$\ell n\, y = \lim\limits_{x\to 0} x\, \ell n\, x$

$= \lim\limits_{x\to 0} - \dfrac{\ell n \frac{1}{x}}{\frac{1}{x}} = 0 \because \quad \dfrac{1}{x} \to \infty \qquad y = 1$

## 10. Sandwich Theorem or Squeeze Play Theorem:

If $f(x) \le g(x) \le h(x)\ \forall\ x$ & $\underset{x\to a}{\text{Limit}}\ f(x) = \ell = \underset{x\to a}{\text{Limit}}\ h(x)$ then $\underset{x\to a}{\text{Limit}}\ g(x) = \ell$.




**Solved Example # 29:** Evaluate $\lim\limits_{n\to\infty} \dfrac{[x]+[2x]+[3x]+....+[nx]}{n^2}$

Where [ ] denotes the greatest integer function.

**Solution.**

We know that, $x - 1 < [x] \le x$

$\Rightarrow \quad 2x - 1 < [2x] \le 2x$

$\Rightarrow \quad 3x - 1 < [3x] \le 3x$

...................

...................

$\Rightarrow \quad nx - 1 < [nx] \le nx$

$\therefore \quad (x + 2x + 3x + .... + nx) - n < [x] + [2x] + ..... + [nx] \le (x + 2x + .... + nx)$

$\Rightarrow \quad \dfrac{xn(n+1)}{2} - n < \sum\limits_{r=1}^{n} [r\, x] \le \dfrac{x.n(n+1)}{2}$

Thus, $\lim\limits_{n\to\infty} \dfrac{[x]+[2x]+....+[nx]}{n^2}$

$\Rightarrow \quad \lim\limits_{n\to\infty} \dfrac{x}{2}\left(1+\dfrac{1}{n}\right) - \dfrac{1}{n} < \lim\limits_{n\to\infty} \dfrac{[x]+[2x]+....+[nx]}{n^2} \le \lim\limits_{n\to\infty} \dfrac{x}{2}\left(1+\dfrac{1}{n}\right)$

$\Rightarrow \quad \dfrac{x}{2} < \lim\limits_{n\to\infty} \dfrac{[x]+[2x]+....+[nx]}{n^2} \le \dfrac{x}{2} \quad \Rightarrow \quad \lim\limits_{n\to\infty} \dfrac{[x]+[2x]+....+[nx]}{n^2} = \dfrac{x}{2}$

***Aliter*** We know that $[x] = x - \{x\}$

$\sum\limits_{r=1}^{n} r\, x = [x] + [2x] + .... + nx - [nx]$

$= (x + 2x + 3x + ... + nx) - (\{x\} + \{2x\} + .... + \{nx\})$

$= \dfrac{xn(n+1)}{2} - (\{x\} + \{2x\} + .. + \{nx\})$

$\therefore \quad \dfrac{1}{n^2}\sum\limits_{r=1}^{n} [r\, x] = \dfrac{x}{2}\left(1+\dfrac{1}{n}\right) - \dfrac{\{x\}+\{2x\}+....+\{nx\}}{n^2}$

Since, $0 \le \{rx\} < 1$, $\quad \therefore \quad 0 \le \sum\limits_{r=1}^{n} [r\, x] < n$

$\Rightarrow \quad \lim\limits_{n\to\infty} \dfrac{\sum\limits_{r=1}^{n}[rx]}{n^2} = 0 \qquad \therefore \qquad \lim\limits_{n\to\infty} \dfrac{\sum\limits_{r=1}^{n}[rx]}{n^2} = \lim\limits_{n\to\infty} \dfrac{x}{2}\left(1+\dfrac{1}{n}\right) - \lim\limits_{n\to\infty} \dfrac{\sum\limits_{r=1}^{n}\{rx\}}{n^2}$

$\lim\limits_{n\to\infty} \dfrac{\sum\limits_{r=1}^{n}[rx]}{n^2} = \dfrac{x}{2}$

**Solved Example # 30** $\underset{x\to 0}{\text{limit}}\ x \sin \dfrac{1}{x}$

**Solution.** $\underset{x\to 0}{\text{limit}}\ x \sin \dfrac{1}{x}$

= 0 × (some value in [– 1, 1]) = 0

## 11. Some Important Notes :

(i) $\lim_{x\to\infty} \frac{\ell nx}{x} = 0$ (ii) $\lim_{x\to\infty} \frac{x}{e^x} = 0$

As $x \to \infty$, $\ell n\, x$ increnes much slower than any (+ve) power of x where $e^x$ increases much faster than (+ve) power of x

(iii) $\text{Limit}_{n\to\infty} (1-h)^n = 0$ & $\text{Limit}_{n\to\infty} (1+h)^n \to \infty$, where h > 0.

(iv) If $\text{Limit}_{x\to a} f(x) = A > 0$ & $\text{Limit}_{x\to a} \phi(x) = B$ (a finite quantity) then;

$\text{Limit}_{x\to a} [f(x)]^{\phi(x)} = e^z$ where $z = \text{Limit}_{x\to a} \phi(x). \ln[f(x)] = e^{B\ln A} = A^B$

**Solved Example # 31** $\lim_{x\to\infty} \frac{x^{1000}}{e^x}$

**Solution.** $\lim_{x\to\infty} \frac{x^{1000}}{e^x} = 0$

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# Short Revesion (LIMIT)

**THINGS TO REMEMBER :**

**1.** Limit of a function f(x) is said to exist as, $x\to a$ when

$$\text{Limit}_{x\to a^-} f(x) = \text{Limit}_{x\to a^+} f(x) = \text{finite quantity.}$$

**2.** **FUNDAMENTAL THEOREMS ON LIMITS :**

Let $\text{Limit}_{x\to a} f(x) = l$ & $\text{Limit}_{x\to a} g(x) = m$. If $l$ & m exists then :

**(i)** $\text{Limit}_{x\to a} f(x) \pm g(x) = l \pm m$ **(ii)** $\text{Limit}_{x\to a} f(x) \cdot g(x) = l.m$

**(iii)** $\text{Limit}_{x\to a} \frac{f(x)}{g(g)} = \frac{\ell}{m}$, provided $m \neq 0$

**(iv)** $\text{Limit}_{x\to a} k f(x) = k\, \text{Limit}_{x\to a} f(x)$ ; where k is a constant.

**(v)** $\text{Limit}_{x\to a} f[g(x)] = f\left(\text{Limit}_{x\to a} g(x)\right) = f(m)$ ; provided f is continuous at g(x) = m.

For example $\text{Limit}_{x\to a} l n(f(x) = l n\left[\text{Limit}_{x\to a} f(x)\right] l n\, l \; (l > 0)$.

**REMEMBER**

$$\text{Limit}_{x\to a} \Rightarrow x \neq a$$

**3.** **STANDARD LIMITS :**

**(a)** $\text{Limit}_{x\to 0} \frac{\sin x}{x} = 1 = \text{Limit}_{x\to 0} \frac{\tan x}{x} = \text{Limit}_{x\to 0} \frac{\tan^{-1} x}{x} = \text{Limit}_{x\to 0} \frac{\sin^{-1} x}{x}$

[ Where x is measured in radians ]

**(b)** $\text{Limit}_{x\to 0} (1+x)^{1/x} = e = \text{Limit}_{x\to\infty} \left(1+\frac{1}{x}\right)^x$ note however there $\text{Limit}_{\substack{h\to 0\\ n\to\infty}} (1-h)^n = 0$

and $\text{Limit}_{\substack{h\to 0\\ n\to\infty}} (1+h)^n \to \infty$

**(c)** If $\text{Limit}_{x\to a} f(x) = 1$ and $\text{Limit}_{x\to a} \phi(x) = \infty$, then ;

$$\text{Limit}_{x\to a} [f(x)]^{\phi(x)} = e^{\text{Limit}_{x\to a} \phi(x)[f(x)-1]}$$

**(d)** If $\text{Limit}_{x\to a} f(x) = A > 0$ & $\text{Limit}_{x\to a} \phi(x) = B$ (a finite quantity) then ;

$\text{Limit}_{x\to a} [f(x)]^{\phi(x)} = e^z$ where $z = \text{Limit}_{x\to a} \phi(x). \ln[f(x)] = e^{B\ln A} = A^B$

**(e)** $\text{Limit}_{x\to 0} \frac{a^x - 1}{x} = 1n\, a \; (a > 0)$. In particular $\text{Limit}_{x\to 0} \frac{e^x - 1}{x} = 1$

**(f)** $\text{Limit}_{x\to a} \frac{x^n - a^n}{x-a} = n a^{n-1}$

**4.** **SQUEEZE PLAY THEOREM :** If $f(x) \le g(x) \le h(x)\ \forall\ x$ & $\text{Limit}_{x\to a} f(x) = l = \text{Limit}_{x\to a} h(x)$ then $\text{Limit}_{x\to a} g(x) = l$.

**5.** **INDETERMINANT FORMS :** $\frac{0}{0}, \frac{\infty}{\infty}, 0\times\infty, 0^\circ, \infty^\circ, \infty-\infty$ and $1^\infty$

**Note : (i)** We cannot plot ∞ on the paper. Infinity (∞) is a symbol & not a number. It does not obey the laws of elementry algebra. **(ii)** $\infty + \infty = \infty$ **(iii)** $\infty \times \infty = \infty$

**(iv)** $(a/\infty) = 0$ if a is finite **(v)** $\frac{a}{0}$ is not defined , if $a \neq 0$.

**(vi)** $ab = 0$ , if & only if a = 0 or b = 0 and a & b are finite.

**6.** The following strategies should be born in mind for evaluating the limits:

**(a)** Factorisation **(b)** Rationalisation or double rationalisation

**(c)** Use of trigonometric transformation ;







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appropriate substitution and using standard limits

**(d)** Expansion of function like Binomial expansion, exponential & logarithmic expansion, expansion of sinx, cosx, tanx should be remembered by heart & are given below :

**(i)** $a^x = 1 + \frac{x\ln a}{1!} + \frac{x^2 \ln^2 a}{2!} + \frac{x^3 \ln^3 a}{3!} + ........ a > 0$

**(ii)** $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + ...........$

**(iii)** $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ........ \text{for } -1 < x \le 1$

**(iv)** $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + .......$

**(v)** $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + ......$

**(vi)** $\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + ........$

**(vii)** $\tan^{-1}x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + .......$

**(viii)** $\sin^{-1}x = x + \frac{1^2}{3!}x^3 + \frac{1^2.3^2}{5!}x^5 + \frac{1^2.3^2.5^2}{7!}x^7 + .......$

**(ix)** $\sec^{-1}x = 1 + \frac{x^2}{2!} + \frac{5x^4}{4!} + \frac{61x^6}{6!} + ......$

# EXERCISE–1

**Q 1.** $\lim_{x\to 1} \frac{x^2 - \sqrt{x}}{\sqrt{x} - 1}$

**Q 2.** $\lim_{x\to 1} \frac{\sqrt[13]{x} - \sqrt[7]{x}}{\sqrt[5]{x} - \sqrt[3]{x}}$

**Q3.** $\lim_{x\to 1} \frac{x^2 - x\ln x + \ln x - 1}{x - 1}$

**Q 4.** $\lim_{x\to 1} \left( \frac{p}{1-x^p} - \frac{q}{1-x^q} \right)$ p, q ∈ N

**Q 5.** $\lim_{x\to\infty} \frac{2\sqrt{x} + 3x^{1/3} + 5x^{1/5}}{\sqrt{3x-2} + (2x-3)^{1/3}}$

**Q 6.** $\lim_{x\to \frac{3\pi}{4}} \frac{1 + \sqrt[3]{\tan x}}{1 - 2\cos^2 x}$

**Q 7.** (a) $\lim_{x\to 0} \tan^{-1} \frac{a}{x^2}$ where $a \in R$

(b) Plot the graph of the function $f(x) = \lim_{t\to 0} \left( \frac{2x}{\pi} \tan^{-1} \frac{x}{t^2} \right)$

**Q 8.** $\lim_{x\to 1} \frac{\left[ \sum_{K=1}^{100} x^k \right] - 100}{x - 1}$

**Q 9.** Find the sum of an infinite geometric series whose first term is the limit of the function

$f(x) = \frac{\tan x - \sin x}{\sin^3 x}$ as $x \to 0$ and whose common ratio is the limit of the function

$g(x) = \frac{1 - \sqrt{x}}{(\cos^{-1} x)^2}$ as $x \to 1$. (Use of series expansion or L'Hospital's rule is not allowed.)

**Q 10.** $\lim_{x\to\infty} (x - \ln \cosh x)$ where $\cosh t = \frac{e^t + e^{-t}}{2}$.

**Q 11.** $\lim_{x\to \frac{\pi}{2}} \cos^{-1}[\cot x]$ where [ ] denotes greatest integer function

**Q 12.** $\lim_{x\to \frac{\pi}{4}} \frac{1 - \tan x}{1 - \sqrt{2}\sin x}$

**Q 13.** $\lim_{x\to 0} [\ln(1 + \sin^2 x). \cot(\ln^2(1 + x))]$

**Q 14.** $\lim_{x\to 0} \frac{\sec 4x - \sec 2x}{\sec 3x - \sec x}$

**Q 15.** $\lim_{\theta\to \frac{\pi}{4}} \frac{\sqrt{2} - \cos\theta - \sin\theta}{(4\theta - \pi)^2}$

**Q 16.** $\lim_{x\to \frac{\pi}{2}} \frac{2^{-\cos x} - 1}{x(x - \frac{\pi}{2})}$

**Q 17.** If $\lim_{x\to 0} \frac{a\sin x - \sin 2x}{\tan^3 x}$ is finite then find the value of 'a' & the limit.

**Q 18.** $\lim_{x\to 0} \frac{8}{x^8}\left[ 1 - \cos\frac{x^2}{2} - \cos\frac{x^2}{4} + \cos\frac{x^2}{2}\cos\frac{x^2}{4} \right]$

**Q 19.** $\lim_{x\to 1} \frac{(\ln(1+x) - \ln 2)(3.4^{x-1} - 3x)}{[(7+x)^{\frac{1}{3}} - (1+3x)^{\frac{1}{2}}].\sin(x-1)}$

**Q 20.** Using Sandwich theorem to evaluate $\lim_{n\to\infty} \left( \frac{1}{\sqrt{n^2}} + \frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + .......... + \frac{1}{\sqrt{n^2+2n}} \right)$

**Q21.** Given $f(x) = \lim_{n\to\infty} \tan^{-1}(nx)$; $g(x) = \lim_{n\to\infty} \sin^{2n} x$ and $\sin(h(x)) = \frac{1}{2}[\cos\pi(g(x)) + \cos(2f(x))]$

Find the domain and range of h (x).

**Q 22.** $\lim_{x\to \frac{\pi}{4}} \frac{\sqrt{1 - \sqrt{\sin 2x}}}{\pi - 4x}$

**Q 23.** $\lim_{x\to 3} \frac{(x^3 + 27)\ln(x - 2)}{x^2 - 9}$

**Q 24.** $\lim_{x\to 2} \frac{(\cos\alpha)^x + (\sin\alpha)^x - 1}{x - 2}$

**Q 25.** $\lim_{x\to 0} \frac{27^x - 9^x - 3^x + 1}{\sqrt{2} - \sqrt{1 + \cos x}}$

**Q 26.** Let $f(x) = \dfrac{x}{\sin x}, \; x > 0$ and $g(x) = x + 3, \quad x < 1$
$= 2 - x, \; x \le 0$ $\quad = x^2 - 2x - 2, \; 1 \le x < 2$
$= x - 5, \quad x \ge 2$
find LHL and RHL of $g(f(x))$ at $x = 0$ and hence find $\lim\limits_{x\to 0} g(f(x))$.

**Q 27.** Let $P_n = a^{P_{n-1}} - 1, \forall n = 2, 3, \ldots\ldots$ and Let $P_1 = a^x - 1$ where $a \in R^+$ then evaluate $\lim\limits_{x\to 0} \dfrac{P_n}{x}$.

**Q 28.** $\lim\limits_{x\to -\infty} \dfrac{(3x^4 + 2x^2)\sin\frac{1}{x} + |x|^3 + 5}{|x|^3 + |x|^2 + |x| + 1}$

**Q.29** If $f(x) = \ln \operatorname{cosec}(x\pi) \quad 0 < x < 1$
$= \ln \sin(2x\pi) \quad 1 < x < 3/2$ and $g(x) = \dfrac{2^{f(x)} + 1}{3^{f(x)} + 1}$ then
find $\tan^{-1}(g(1^-))$ and $\sec^{-1}(g(1^+))$.

**Q.30** At the end-points and the midpoint of a circular arc AB tangent lines are drawn, and the points A and B are joined with a chord. Prove that the ratio of the areas of the two triangles thus formed tends to 4 as the arc AB decreases indefinitely.

# EXERCISE–2

**Q 1.** $\lim\limits_{x\to\infty} \left[\dfrac{2x^2+3}{2x^2+5}\right]^{8x^2+3}$

**Q 2.** $\lim\limits_{x\to\infty} \left(\dfrac{x+c}{x-c}\right)^x = 4$ then find c

**Q 3.** $\lim\limits_{x\to 0} \left[\dfrac{(1+x)^{1/x}}{e}\right]^{1/x}$

**Q 4.** $\lim\limits_{x\to 0} \left[\sin^2\left(\dfrac{\pi}{2-ax}\right)\right]^{\sec^2\left(\frac{\pi}{2-bx}\right)}$

**Q.5** $\lim\limits_{x\to\infty} x^2 \sin\left(\ell n \sqrt{\cos\dfrac{\pi}{x}}\right)$

**Q 6.** $\lim\limits_{x\to\infty} \left[\cos\left(2\pi\left(\dfrac{x}{1+x}\right)^a\right)\right]^{x^2} \quad a \in R$

**Q 7.** $\lim\limits_{x\to 1} \left(\tan\dfrac{\pi x}{4}\right)^{\tan\frac{\pi x}{2}}$

**Q 8.** $\lim\limits_{x\to 0} \left(\dfrac{x - 1 + \cos x}{x}\right)^{\frac{1}{x}}$

**Q 9.** $\lim\limits_{x\to\infty} \left(\dfrac{a_1^{\frac{1}{x}} + a_2^{\frac{1}{x}} + a_3^{\frac{1}{x}} + \ldots + a_n^{\frac{1}{x}}}{n}\right)^{nx}$ where $a_1, a_2, a_3, \ldots\ldots a_n > 0$

**Q 10.** Let $f(x) = \dfrac{\sin^{-1}(1-\{x\}).\cos^{-1}(1-\{x\})}{\sqrt{2\{x\}}\,.\,(1-\{x\})}$ then find $\lim\limits_{x\to 0^+} f(x)$ and $\lim\limits_{x\to 0^-} f(x)$, where {x} denotes the fractional part function.

**Q 11.** Find the values of a, b & c so that $\lim\limits_{x\to 0} \dfrac{ae^x - b\cos x + ce^{-x}}{x.\sin x} = 2$

**Q 12.** $\lim\limits_{x\to a} \dfrac{1}{(a^2 - x^2)^2} \left(\dfrac{a^2 + x^2}{ax} - 2\sin\left(\dfrac{a\pi}{2}\right)\sin\left(\dfrac{\pi x}{2}\right)\right)$ where a is an odd integer

**Q 13.** $\lim\limits_{x\to 0} \dfrac{\tan^2 x - x^2}{x^2 \tan^2 x}$

**Q 14.** $\lim\limits_{n\to\infty} \dfrac{x^n f(x) + g(x)}{x^n + 1} \quad x \,\varepsilon\, R$

**Q 15.** $\lim\limits_{n\to\infty} \dfrac{[1.x] + [2.x] + [3.x] + \ldots + [n.x]}{n^2}$, Where [.] denotes the greatest integer function.

**Q 16.** Without using series expansion or L'Hospital's rule evaluate, $\lim\limits_{x\to 1} \dfrac{1 - x + \ln x}{1 + \cos \pi x}$

**Q 17.** $\lim\limits_{y\to 0} \left[\operatorname{Limit}\limits_{x\to\infty} \dfrac{\exp\left(x\ln(1+\frac{ay}{x})\right) - \exp\left(x\ln(1+\frac{by}{x})\right)}{y}\right]$

**Q 18.** If $s_n$ be the sum of n terms of the series, sin x + sin 2x + sin 3x +..... + sin nx then show that $\operatorname{Limit}\limits_{n\to\infty} \dfrac{s_1 + s_2 + \ldots\ldots + s_n}{n} = \dfrac{1}{2}\cot\dfrac{x}{2} \quad (x \ne 2k\pi, \; k \in I)$

**Q 19.** $\lim\limits_{x\to 0} \left[\dfrac{\ell n (1+x)^{1+x}}{x^2} - \dfrac{1}{x}\right]$

**Q 20.** Let $P_n = \dfrac{2^3-1}{2^3+1}.\dfrac{3^3-1}{3^3+1}.\dfrac{4^3-1}{4^3+1}\ldots\ldots\dfrac{n^3-1}{n^3+1}$. Evaluate $\lim\limits_{n\to\infty} P_n$

**Q 21.** A circular arc of radius 1 subtends an angle of x radians, $0 < x < \dfrac{\pi}{2}$ as shown in the figure. The point C is the intersection of the two tangent lines at A & B. Let T(x) be the area of triangle ABC & let S(x) be the area of the shaded region. Compute :



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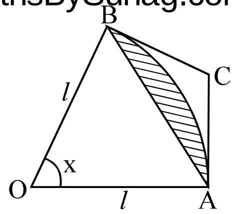



(a) T(x) (b) S(x) & (c) the limit of $\frac{T(x)}{S(x)}$ as $x \to 0$.

**Q 22.** **(a)** $\lim_{x\to\infty} \frac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}}$ **(b)** $\lim_{x\to\infty}\left[\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right]$

**Q 23.** If $f(n,\theta)=\prod_{r=1}^{n}\left(1-\tan^2\frac{\theta}{2^r}\right)$, then compute $\lim_{n\to\infty} f(n,\theta)$

**Q 24.** Let $l = \lim_{x\to a}\frac{x^x - a^x}{x-a}$ & $m = \lim_{x\to a}\frac{a^x - x^a}{x-a}$ where a > 0. If $l$ = m then find the value of 'a'.

**Q 25.** $\lim_{x\to\infty}\left(\frac{\cosh\frac{\pi}{x}}{\cos\frac{\pi}{x}}\right)^{x^2}$ where $\cosh t = \frac{e^t + e^{-t}}{2}$ **Q 26.** $\lim_{x\to 0}\frac{2(\tan x - \sin x) - x^3}{x^5}$

**Q 27.** Through a point A on a circle, a chord AP is drawn & on the tangent at A a point T is taken such that AT = AP. If TP produced meet the diameter through A at Q, prove that the limiting value of AQ when P moves upto A is double the diameter of the circle.

**Q 28.** Using Sandwich theorem, evaluate

**(a)** $\lim_{n\to\infty}\frac{1}{1+n^2}+\frac{2}{2+n^2}+\dots\dots\dots+\frac{n}{n+n^2}$ **(b)** $\lim_{n\to\infty}(a^n+b^n)^{\frac{1}{n}}$, 0 < a < b

**Q 29.** Find a & b if : **(i)** $\lim_{x\to\infty}\left[\frac{x^2+1}{x+1}-ax-b\right]=0$ **(ii)** $\lim_{x\to-\infty}\left[\sqrt{x^2-x+1}-ax-b\right]=0$

**Q 30.** Show that $\lim_{h\to 0}\frac{(\sin(x+h))^{x+h}-(\sin x)^x}{h} = (\sin x)^x\,[\,x\cot x + l\,n \sin x\,]$

# EXERCISE–3

Q.1 $\lim_{x\to 0}\left[\frac{1+5x^2}{1+3x^2}\right]^{\frac{1}{x^2}}$ = ______. [ IIT'96, 1 ]

Q.2 $\lim_{x\to 1}\frac{\sqrt{1-\cos 2(x-1)}}{x-1}$ [ IIT '98, 2 ]

(A) exits and it equals $\sqrt{2}$ (B) exits and it equals - $\sqrt{2}$
(C) does not exist because x - 1 → 0
(D) does not exist because left hand limit is not equal to right hand limit.

Q.3 $\lim_{x\to 0}\frac{x\tan 2x - 2x\tan x}{(1-\cos 2x)^2}$ is : [ JEE '99, 2 (out of 200) ]

(A) 2 (B) –2 (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

Q.4 For $x \in R$, $\lim_{x\to\infty}\left(\frac{x-3}{x+2}\right)^x$ = [ JEE 2000, Screening]

(A) e (B) $e^{-1}$ (C) $e^{-5}$ (D) $e^5$

Q.5 $\lim_{x\to 0}\frac{\sin(\pi\cos^2 x)}{x^2}$ equals [ JEE 2001, Screening]

(A) –π (B) π (C) $\frac{\pi}{2}$ (D) 1

Q.6 Evaluate $\lim_{x\to 0}\frac{a^{\tan x}-a^{\sin x}}{\tan x - \sin x}$, a > 0. [REE 2001, 3 out of 100]

Q.7 The integer n for which $\lim_{x\to 0}\frac{(\cos x - 1)(\cos x - e^x)}{x^n}$ is a finite non-zero number is

(A) 1 (B) 2 (C) 3 (D) 4

[JEE 2002 (screening), 3]

Q.8 If $\lim_{x\to 0}\frac{\sin(nx)[(a-n)nx-\tan x]}{x^2}=0$ (n > 0) then the value of 'a' is equal to



(A) $\frac{1}{n}$ (B) $n^2 + 1$ (C) $\frac{n^2+1}{n}$ (D) None

[JEE 2003 (screening)]

Q.9 Find the value of $\lim_{n\to\infty}\left[\frac{2}{\pi}(n+1)\cos^{-1}\left(\frac{1}{n}\right)-n\right]$. [ JEE ' 2004, 2 out of 60]

# EXERCISE–4

**1.** $\underset{n\to\infty}{\text{Limit}} \frac{5^{n+1}+3^n-2^{2n}}{5^n+2^n+3^{2n+3}}$ = (A) 5 (B) 3 (C) 1 (D) zero

**2.** $\underset{x\to -1}{\text{Limit}} \frac{\cos 2-\cos 2x}{x^2-|x|}$ =

(A) 2 cos 2 (B) – 2 cos 2 (C) 2 sin 2 (D) – 2 sin 2

**3.** The value of $\underset{x\to 0}{\text{Limit}} \frac{1}{x}\sqrt{\frac{1-\cos 2x}{2}}$ is:

(A) 1 (B) – 1 (C) 0 (D) none

**4.** $\underset{x\to 0}{\text{Limit}} \sin^{-1}(\sec x)$.

(A) is equal to $\pi/2$ (B) is equal to 1 (C) is equal to zero (D) none of these

**5.** $\underset{x\to 5}{\text{Limit}} \frac{x^2-9x+20}{x-[x]}$ where [x] is the greatest integer not greater than x:

(A) is equal to 1 (B) 0 (C) 4 (D) none

**6.** $\underset{x\to -\pi}{\text{Limit}} \frac{|x+\pi|}{\sin x}$:

(A) is equal to – 1 (B) is equal to 1 (C) is equal to $\pi$ (D) does not exist

**7.** $\underset{x\to 3}{\text{Limit}} \frac{(x^3+27)\ln(x-2)}{(x^2-9)}$ =

(A) – 8 (B) 8 (C) 9 (D) – 9

**8.** $\underset{x\to 1}{\text{Limit}} \frac{\sum_{k=1}^{100} x^k - 100}{x-1}$ =

(A) 0 (B) 5050 (C) 4550 (D) – 5050

**9.** $\underset{x\to\infty}{\text{Limit}} \left(\sqrt{(x+a)(x+b)}-x\right)$ =

(A) $\sqrt{ab}$ (B) $\frac{a+b}{2}$ (C) ab (D) none

**10.** $\underset{x\to\infty}{\text{Limit}} \frac{x^3 \cdot \sin\frac{1}{x}+x+1}{x^2+x+1}$ =

(A) 0 (B) 1/2 (C) 1 (D) none

**11.** $\underset{n\to\infty}{\text{Limit}} \frac{(n+2)!+(n+1)!}{(n+3)!}$, $n \in N$ =

(A) 0 (B) 1 (C) 2 (D) – 1

**12.** $\underset{x\to 0}{\text{Limit}} |x|^{\sin x}$ =

(A) 0 (B) 1 (C) – 1 (D) none of these

**13.** $\underset{x\to\infty}{\text{Limit}} \left(\frac{x^2-2x+1}{x^2-4x+2}\right)^x$ =

(A) 1 (B) 2 (C) $e^2$ (D) e

**14.** The values of a and b such that $\lim_{x\to 0}\frac{x(1+a\cos x)-b\sin x}{x^3} = 1$ are

(A) $\frac{5}{2}, \frac{3}{2}$ (B) $\frac{5}{2}, -\frac{3}{2}$ (C) $-\frac{5}{2}, -\frac{3}{2}$ (D) $-\frac{5}{2}, \frac{3}{2}$

**15.** $\underset{x\to 0}{\text{Limit}} \frac{2\left(\sqrt{3}\sin\left(\frac{\pi}{6}+x\right)-\cos\left(\frac{\pi}{6}+x\right)\right)}{x\sqrt{3}\left(\sqrt{3}\cos x-\sin x\right)}$ =

(A) – 1/3 (B) 2/3 (C) 4/3 (D) – 4/3

**16.** If $f(x) = \begin{cases} x-1, & x \ge 1 \\ 2x^2-2, & x < 1 \end{cases}$, $g(x) = \begin{cases} x+1, & x > 0 \\ -x^2+1, & x \le 0 \end{cases}$ and $h(x) = |x|$

then find $\lim_{x\to 0} f(g(h(x)))$

(A) 1 (B) 0 (C) –1 (D) does not exists

**17.** $\text{Limit}_{x\to 1} (1 - x + [x - 1] + [1 - x])$ = where [x] denotes greatest integer function.
(A) 0 (B) 1 (C) $-1$ (D) does not exist

**18.** $\text{Lt}_{x\to 0} \left[\dfrac{\sin[x-3]}{[x-3]}\right]$, where [ . ] denotes greatest integer function is :
(A) 0 (B) 1 (C) does not exist (D) sin 1

**19.** Let $f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right)+\sin\left(\frac{1}{x^2}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$, then $\lim_{x\to\infty} f(x)$ equals
(A) 0 (B) $-1/2$ (C) 1 (D) none of these.

**20.** $\lim_{x\to a^-}\left(\dfrac{|x|^3}{a} - \left[\dfrac{x}{a}\right]^3\right)$ (a > 0), where [x] denotes the greatest integer less than or equal to x is
(A) $a^2 + 1$ (B) $a^2 - 1$ (C) $a^2$ (D) $-a^2$

**21.** Let $\alpha$, $\beta$ be the roots of $ax^2 + bx + c = 0$, where $1 < \alpha < \beta$. Then $\text{Lim}_{x\to x_0} \dfrac{|ax^2+bx+c|}{ax^2+bx+c} = 1$ then which of the following statements is incorrect
(A) $a > 0$ and $x_0 < 1$ (B) $a > 0$ and $x_0 > \beta$
(C) $a < 0$ and $\alpha < x_0 < \beta$ (D) $a < 0$ and $x_0 < 1$

**22.** $\text{Limit}_{n\to\infty} \dfrac{1n+2(n-1)+3(n-2)+......+n1}{1^2+2^2+3^2+......+n^2}$ has the value :
(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) 1

**23.** $\text{Lim}_{x\to 0}\left[(1-e^x)\dfrac{\sin x}{|x|}\right]$ is (where [·] represents greatest integral part function)
(A) $-1$ (B) 1 (C) 0 (D) does not exist

**24.** If $\ell = \lim_{x\to\infty} (\sin\sqrt{x+1} - \sin\sqrt{x})$ and $m = \lim_{x\to-\infty} [\sin\sqrt{x+1} - \sin\sqrt{x}]$ where [.] denotes the greatest integer function then :
(A) $\ell = m = 0$ (B) $\ell = 0$ ; m is undefined
(C) $\ell$, m both do not exist (D) $\ell = 0$, $m \neq 0$ (although m exist)

**25.** If $f(x) = \sum_{\lambda=1}^{n}\left(x - \frac{1}{\lambda}\right)\left(x - \frac{1}{\lambda+1}\right)$ then $\lim_{n\to\infty} f(0)$ is.
(A) 1 (B) $-1$ (C) 2 (D) None

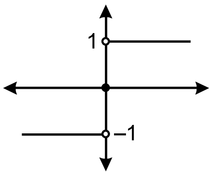
**26.** The limit $\lim_{\theta\to 0}\left(\left[\dfrac{n\sin\theta}{\theta}\right] + \left[\dfrac{n\tan\theta}{\theta}\right]\right)$, where [x] is the greatest integer function and $n \in I$, is
(A) 2n (B) 2n + 1 (C) 2n – 1 (D) does not exist

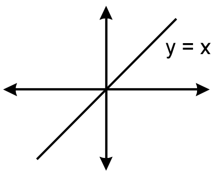
**27.** The limit $\text{Limit}_{x\to\infty} x - x^2\,\ell n\left(1 + \frac{1}{x}\right)$ is equal to :
(A) 1/2 (B) 3/2 (C) 1/3 (D) 1

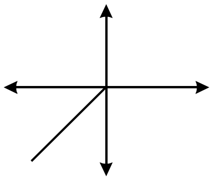
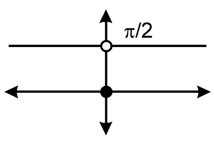
**28.** $\lim_{x\to\pi/2}\left[\dfrac{x - \frac{\pi}{2}}{\cos x}\right]$ is : (where [.] represents greatest integer function.
(A) $-1$ (B) 0 (C) $-2$ (D) does not exist

**29.** If f(x) = sin x , $x \neq n\pi$ , n = 0 , ± 1 , ± 2 , ± 3, ,....
= 2 , otherwise and
g(x) = $x^2 + 1$ , $x \neq 0$ , 2
= 4 , x = 0
= 5 , x = 2
then $\text{Limit}_{x\to 0}$ g [f(x)] is :
(A) 1 (B) 0 (C) 4 (D) does not exists

**30.** The graph of the function $f(x) = \text{Lim}_{t\to 0}\left(\dfrac{2x}{\pi}\cot^{-1}\dfrac{x}{t^2}\right)$, is

(A) 

(B) 

(C) 
(D) 


**31.** The value of $\text{Limit}_{x\to 0} \dfrac{\cos(\sin x) - \cos x}{x^4}$ is equal to:
(A) 1/5 (B) 1/6 (C) 1/4 (D) 1/2

**32.** $\underset{x\to\infty}{\text{Limit}} \dfrac{e^x\left(\left(2^{x^n}\right)^{\frac{1}{e^x}} - \left(3^{x^n}\right)^{\frac{1}{e^x}}\right)}{x^n}$, n ∈ N is equal to :

(A) 0 (B) ln(2/3) (C) ln(3/2) (D) none

**33.** $\underset{y\to 0}{\text{Limit}}\left(\underset{x\to\infty}{\text{Limit}}\dfrac{\exp\left(x\ln\left(1+\frac{ay}{x}\right)\right)-\exp\left(x\ln\left(1+\frac{by}{x}\right)\right)}{y}\right) =$

(A) a + b (B) a – b (C) b – a (D) – (a + b)

# EXERCISE–5

**1.** Evaluate the following limits, where [ . ] represents greatest integer function and { . } represents fractional part function

(i) $\lim\limits_{x\to\frac{\pi}{2}} [\sin x]$ (ii) $\lim\limits_{x\to 2}\left\{\frac{x}{2}\right\}$ (iii) $\lim\limits_{x\to\pi} \text{sgn}\,[\tan x]$

**2.** If f(x) = $\begin{cases} x^2+2 & , \ x \ge 2 \\ 1-x & , \ x < 2 \end{cases}$ and g(x) = $\begin{cases} 2x & , \ x > 1 \\ 3-x & , \ x \le 1 \end{cases}$, evaluate $\lim\limits_{x\to 1} f(g(x))$.

**3.** Evaluate each of the following limits, if exists

(i) $\lim\limits_{x\to 2}\dfrac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}}$ (ii) $\lim\limits_{x\to a}\dfrac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}$, a ≠ 0

**4.** Evaluate the following limits, if exists

(i) $\lim\limits_{x\to 0}\dfrac{\tan 3x-2x}{3x-\sin^2 x}$ (ii) $\lim\limits_{x\to a}\dfrac{(x+2)^{\frac{5}{2}}-(a+2)^{\frac{5}{2}}}{x-a}$ (iii) $\lim\limits_{x\to 0}\dfrac{x\left(e^{2+x}-e^2\right)}{1-\cos x}$

**5.** Evaluate the following limits, if exist :

(i) $\lim\limits_{x\to\infty}\sqrt{x^2+x-1}-x$ (ii) $\lim\limits_{x\to\infty}\left(\frac{1}{x^2}+\frac{2}{x^2}+....+\frac{x}{x^2}\right)$

(iii) $\underset{x\to\infty}{\text{Limit}}\left\{\cos\left(\sqrt{x+1}\right)-\cos\left(\sqrt{x}\right)\right\}$ (iv) $\lim\limits_{x\to-\infty}\sqrt{x^2-8x}+x$

**6.** Evaluate the following limits using expansions : (i) $\lim\limits_{x\to 0}\dfrac{e^x-1-\sin x-\frac{\tan^2 x}{2}}{x^3}$

(ii) If $\lim\limits_{x\to 0}\dfrac{a+b\sin x-\cos x+ce^x}{x^3}$ exists, then find values of a, b, c. Also find the limit

**7.** Evaluate $\lim\limits_{x\to\infty}\dfrac{[1.\,2x]+[2.\,3x]+.....+[n.\,(n+1)\,x]}{n^3}$ where [ . ] denotes greatest integer function

**8.** If f(x) = $\underset{n\to\infty}{\text{Lim}}\dfrac{x^{2n}-1}{x^{2n}+1}$, find range of f(x).

**9.** Evaluate the following limits

(i) $\underset{x\to 1}{\text{Limit}}\dfrac{(\ln(1+x)-\ln 2)(3.4^{x-1}-3x)}{[(7+x)^{\frac{1}{3}}-(1+3x)^{\frac{1}{2}}].\sin(x-1)}$ (ii) $\underset{x\to 4}{\text{Limit}}\dfrac{(\cos\alpha)^x-(\sin\alpha)^x-\cos 2\alpha}{x-4}$, $\alpha\in\left(0,\frac{\pi}{2}\right)$

**10.** Evaluate the following limits

(i) $\underset{x\to\infty}{\text{Limit}}\ x^3\left\{\sqrt{x^2+\sqrt{1+x^4}}-x\sqrt{2}\right\}$ (ii) $\underset{x\to-\infty}{\text{Limit}}\dfrac{x^5\tan\left(\frac{1}{\pi x^2}\right)+3|x|^2+7}{|x|^3+7|x|+8}$

**11.** Evaluate the following limits (i) $\underset{x\to 0}{\text{Limit}}\left[\sin^2\left(\dfrac{\pi}{2-ax}\right)\right]^{\sec^2\left(\frac{\pi}{2-bx}\right)}$

(ii) $\underset{x\to\infty}{\text{Limit}}\left(\dfrac{a_1^{1/x}+a_2^{1/x}+a_3^{1/x}+.......+a_n^{1/x}}{n}\right)^{nx}$, where $a_1, a_2, a_3, ......., a_n > 0$.

**12.** Find the values of a & b so that: (i) $\underset{x\to 0}{\text{Limit}}\dfrac{(1+ax\sin x)-(b\cos x)}{x^4}$ may find to a definite limit.

(ii) $\lim\limits_{x\to\infty}\left(\sqrt{x^4+ax^3+3x^2+bx+2}-\sqrt{x^4+2x^3-cx^2+3x-d}\right) = 4$

**13.** Find the limits using expansion : $\underset{x\to 0}{\text{Limit}}\left[\dfrac{\ell n(1+x)^{(1+x)}}{x^2}-\dfrac{1}{x}\right]$

**14.** Let $f(x) = \dfrac{\sin^{-1}(1-\{x\}).\cos^{-1}(1-\{x\})}{\sqrt{2\{x\}}.(1-\{x\})}$ then find $\underset{x\to 0^+}{\text{Limit}}\ f(x)$ and $\underset{x\to 0^-}{\text{Limit}}\ f(x)$, where {.} denotes the fractional part function.

**15.** Let $f(x) = \underset{m\to\infty}{\text{Limit}}\left\{\underset{n\to\infty}{\text{Limit}}\left(\cos^{2m}(n!\pi x)\right)\right\}$ where $x \in R$. Prove that

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}.$$

**16.** Evaluate $\underset{x\to 0^+}{\text{Limit}}\left\{\underset{n\to\infty}{\text{Limit}}\left(\dfrac{[1^2(\sin x)^x]+[2^2(\sin x)^x]+.......+[n^2(\sin x)^x]}{n^3}\right)\right\}$,

where [. ] denotes the greatest integer function.

**17.** Evaluate the following limits

(i) $\underset{n\to\infty}{\text{Limit}}\ \cos\dfrac{x}{2}\ \cos\dfrac{x}{4}\ \cos\dfrac{x}{8}........\cos\dfrac{x}{2^n}$

(ii) $\underset{n\to\infty}{\text{Limit}}\ \dfrac{1}{2}\tan\dfrac{x}{2} + \dfrac{1}{2^2}\tan\dfrac{x}{2^2} + \dfrac{1}{2^3}\tan\dfrac{x}{2^3} + ...... + \dfrac{1}{2^n}\tan\dfrac{x}{2^n}$.

(iii) $\underset{x\to\infty}{\text{Limit}}\ \log_{x-1}(x).\log_x(x+1).\log_{x+1}(x+2).\log_{x+2}(x+3).....\log_k(x^5)$ ; where $k = x^5 - 1$.

(iv) Let $P_n = \dfrac{2^3-1}{2^3+1}.\dfrac{3^3-1}{3^3+1}.\dfrac{4^3-1}{4^3+1}..........\dfrac{n^3-1}{n^3+1}$. Prove that $\underset{n\to\infty}{\text{Limit}}\ P_n = \dfrac{2}{3}$.

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## ANSWER

### EXERCISE–1

**Q 1.** 3 **Q 2.** $\dfrac{45}{91}$ **Q 3.** 2 **Q 4.** $\dfrac{p-q}{2}$ **Q 5.** $\dfrac{2}{\sqrt{3}}$ **Q.6** $-\dfrac{1}{3}$

**Q 7.** (a) $\pi/2$ if $a > 0$ ; 0 if $a = 0$ and $-\pi/2$ if $a < 0$ (b) $f(x) = |x|$

**Q 8.** 5050 **Q 9.** $a = \dfrac{1}{2}$; $r = \dfrac{1}{4}$; $S = \dfrac{2}{3}$ **Q 10.** $l$n 2 **Q 11.** does not exist **Q 12.** 2

**Q 13.** 1 **Q 14.** $\dfrac{3}{2}$ **Q 15.** $\dfrac{1}{16\sqrt{2}}$ **Q 16.** $\dfrac{21n2}{\pi}$

**Q 17.** a = 2 ; limit = 1 **Q 18.** $\dfrac{1}{32}$ **Q 19.** $-\dfrac{9}{4}\ 1n\ \dfrac{4}{e}$ **Q 20.** 2

**Q 21.** Domain, $x \in R$, Range, $x = \dfrac{n\pi}{2}$, $n \in I$ **Q 22.** does not exist **Q 23.** 9

**Q 24.** $\cos^2\alpha\ l\text{n}\text{Cos}\alpha + \text{Sin}^2\alpha\ l\text{n}\text{Sin}\alpha$ **Q 25.** $8\sqrt{2}(1n3)^2$ **Q 26.** – 3, –3, – 3

**Q 27.** $(l\text{n } a)^n$ **Q 28.** –2 **Q 29.** 0, 0 **Q. 30** 4

### EXERCISE–2

**Q 1.** $e^{-8}$ **Q 2.** c = ln2 **Q 3.** $e^{-\frac{1}{2}}$ **Q 4.** $e^{-a^2/b^2}$ **Q.5** $-\dfrac{\pi^2}{4}$ **Q 6.** $e^{-2\pi^2 a^2}$ **Q 7.** $e^{-1}$

**Q 8.** $e^{-1/2}$ **Q 9.** $(a_1.a_2.a_3....a_n)$ **Q 10.** $\dfrac{\pi}{2}, \dfrac{\pi}{2\sqrt{2}}$ **Q 11.** a = c = 1, b = 2 **Q 12.** $\dfrac{\pi^2a^2+4}{16a^4}$

**Q 13.** $\dfrac{2}{3}$ **Q 14.** f(x) when $|x| > 1$ ; g(x) when $|x| < 1$ ; $\dfrac{g(x)+f(x)}{2}$ when x = 1 & not defined when x = -1

**Q 15.** $\dfrac{x}{2}$ **Q 16.** $-\dfrac{1}{\pi^2}$ **Q 17.** a - b **Q 19.** 1/2 **Q 20.** $\dfrac{2}{3}$

**Q 21.** $T(x) = \dfrac{1}{2}\tan^2\dfrac{x}{2}.\sin x$ or $\tan\dfrac{x}{2} - \dfrac{\sin x}{2}$, $S(x) = \dfrac{1}{2}x - \dfrac{1}{2}\sin x$, limit $= \dfrac{3}{2}$

**Q 22.** **(a)** 1 **(b)** $\dfrac{1}{2}$ **Q 23.** $\dfrac{\theta}{\tan\theta}$ **Q 24.** $a = e^2$ **Q 25.** $e^{\pi^2}$

**Q 26.** $\dfrac{1}{4}$ **Q 28.** (a) 1/2, (b) b **Q 29.** **(i)** a =1, b = –1 **(ii)** a = –1 , b = $\dfrac{1}{2}$

### EXERCISE–3

**Q 1.** $e^2$ **Q 2.** D **Q 3.** C **Q 4.** C **Q5.** B

**Q6.** $l$na **Q7.** C **Q8.** C **Q.9** $1-\dfrac{2}{\pi}$

## EXERCISE–4

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** D | **2.** C | **3.** D | **4.** D | **5.** D | **6.** D | **7.** C | **8.** B | **9.** B | **10.** C |
| **11.** A | **12.** B | **13.** C | **14.** C | **15.** C | **16.** B | **17.** C | **18.** C | **19.** C | **20.** C |
| **21.** D | **22.** A | **23.** A | **24.** B | **25.** A | **26.** C | **27.** A | **28.** C | **29.** A | **30.** C |
| **31.** B | **32.** B | **33.** B | | | | | | | |

## EXERCISE–5

**1.** (i) 0 (ii) Limit does not exists (iii) Limit does not exists

**2.** 6 **3.** (i) (– 8) (ii) $\frac{2}{3\sqrt{3}}$ **4.** (i) 1/3 (ii) $\frac{5}{2}(a+2)^{3/2}$ (iii) $2e^2$

**5.** (i) 1/2 (ii) 1/2 (iii) zero (iv) $\infty$ **6.** (i) $\frac{1}{3}$ (ii) a = 2, b = 1, c = –1 and value = $-\frac{1}{3}$

**7.** $\frac{x}{3}$ **8.** {–1, 0, 1} **9.** *(i)* $-\frac{9}{4}\ln\frac{4}{e}$ *(ii)* $\cos^4 a \ln(\cos a) - \sin^4 a \ln(\sin a)$

**10.** (i) $\frac{1}{4\sqrt{2}}$ (ii) $-\frac{1}{\pi}$ **11.** (i) $e^{-\frac{a^2}{b^2}}$ (ii) $(a_1 a_2 a_3 \ldots\ldots a_n)$

**12.** (i) $a = -\frac{1}{2}$, b = 1 (ii) a = 2, $b \in R$, c = 5, $d \in R$ **13.** $\frac{1}{2}$

**14.** $\frac{\pi}{2}, \frac{\pi}{2\sqrt{2}}$ **16.** $\frac{1}{3}$ **17.** (i) $\frac{\sin x}{x}$ (ii) $\frac{1}{x} - \cot x$ (iii) 5





 